TECHNICAL NOTE

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DEFLECTION OF SUSPENDED PIPES

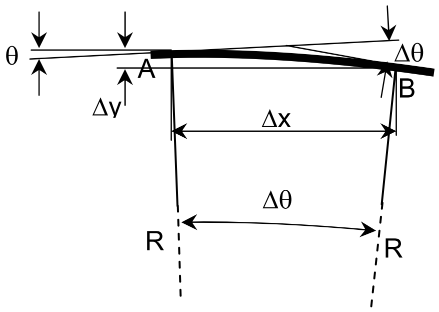
Introduction

Pipes suspended above ground develop vertical deflection between supports. Usually it is desirable to limit this for aesthetic reasons, but there may be practical reasons also, for example to avoid settling of solids or pooling of liquids in drainage lines.

The first criterion involves knowledge of the centre-span deflection of the pipe, and an arbitary judgement as to the acceptable value. The second requires prediction of the maximum slope of the deflected pipe to ensure that it does not fall below an acceptable gradient, usually zero.

This note demonstrates the methods of calculation and suggests appropriate design values.

General theory



For a small section of beam Δx[1]

$$\Delta\theta = \frac{\Delta x}{R_x} \qquad \text{..........................Eq 1}$$

$$\Delta y = \theta\Delta x \qquad \text{..........................Eq 2}$$

where R_x is the radius of curvature at position x along the beam, generated by the bending moment at that point M_x according to the relationship

$$R_x = \frac{EI}{M_x} \qquad \text{..........Eq 3}$$

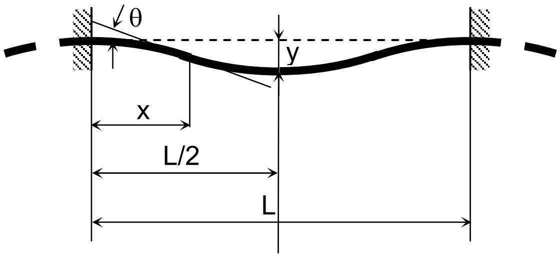
Combining Equations 1 and 3 and integrating along the beam enables the change in slope between any two sections to be calculated:

$$\theta = \int_A^B \frac{1}{R(x)}dx = \frac{1}{EI}\int_A^B M(x)\,dx \qquad \text{....Eq 4}$$

Integrating again the slope along the beam gives the change in vertical deflection between the two sections.

$$y = \int_A^B \theta\,dx \qquad \text{....Eq 5}$$

Internal spans



For the case of a beam with fixed supports and a uniformly distributed load above, which simulates a continuous pipe on point supports, the bending moment function of x is:

[1] It is accepted in beam theory that distance around the deflected beam is sensibly the same as the distance along the undeflected beam

$$M(x) = M_A - \frac{wLx}{2} + \frac{wx^2}{2}$$

$$= \frac{wL^2}{12} - \frac{wLx}{2} + \frac{wx^2}{2}$$ [2]

$$= \frac{w}{12}\left[L^2 - 6Lx + 6x^2\right]$$

Substituting into 4:

$$\theta(x) = \frac{w}{12EI}\int_0^x \left[L^2 - 6xL + 6x^2\right]dx$$

$$= \frac{w}{12EI}\left[L^2x - 3Lx^2 + 2x^3\right] \quad \text{....Eq 6}$$

The maximum slope occurs at the point of inflection, where the bending moment (first derivative of slope) is zero:

$$L^2 - 6Lx + 6x^2 = 0$$

$$x = \frac{6L \pm \sqrt{36L^2 - 24L^2}}{12}$$

$$= \left(\frac{1}{2} \pm \frac{1}{2\sqrt{3}}\right)L = 0.5L \pm 0.289L$$

Substituting one solution back into equation 6:

$$\theta_{max} = \frac{wL^3}{12EI}\left[0.211 - (.211)^2 + (.211)^3\right]$$

$$= \frac{0.0147wL^3}{EI} \quad \text{....Eq 7}$$

Integrating θ from equation 6:

$$y(x) = \frac{w}{12EI}\int_0^x \left[L^2x - 3Lx^2 + 2x^3\right]dx$$

$$= \frac{w}{12EI}\left[\frac{L^2x^2}{2} - Lx^3 + \frac{x^4}{2}\right] \text{....Eq 8}$$

The maximum deflection occurs at centre-span where x = L/2:

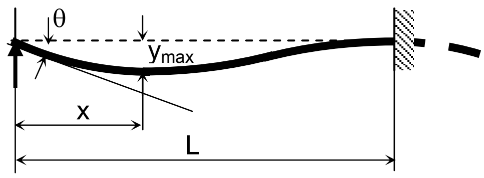
$$y_{max} = \frac{wL^4}{12EI}\left[\frac{1}{8} - \frac{1}{8} + \frac{1}{32}\right]$$

$$= \frac{wL^4}{384EI} \quad \text{....Eq 9}$$

End spans

A further case of significance occurs at the end of a pipe run, where the end is free to rotate, as above. (The simulation using a fixed end on the right hand end is an approximation, since some rotation occurs at that support also).

[2] The formula for M_A at the left hand support is not derived here since it is available in standard texts. See "Roark's Formulas for Stress & Strain", WC Young (Ed), McGraw-Hill



By similar analysis, the following equations can be derived:

$$\theta_{max} = \frac{wL^3}{48EI} \quad \text{at } x = 0 \text{ (LH end)} \quad \text{...Eq 10}$$

$$y_{max} = 0.0054\frac{wL^4}{EI} \quad \text{at } x = 0.4215L \text{ ...Eq 11}$$

Example:

DN 50 DWV AS1260 PVC-U pipe at 2 m span, 40°C repetitive short term discharge.
I = 1.39E+05 mm^4
w = 0.0028x9.8 N/mm
E = 700 N/mm^2
L = 2000 mm

Internal span:
θ_{max} = 0.033 radians = 1.9° = 3.3%
y_{max} = 12 mm
End span
θ_{max} = 0.047 radians = 2.7° = 4.7%
y_{max} = 24 mm
By reducing the end-span to 1.67 m, the deflections are reduced to values similar to the internal spans:
θ_{max} = 0.027 radians = 1.5° = 2.7%
y_{max} = 12 mm

Modulus values

The analysis above is based on elastic beam theory. It may be applied to visco-elastic materials by use of the creep modulus appropriate to the time and temperature of loading. It should be borne in mind that part of the visco-elastic response is irrecoverable, and repeated short term events may have cumulative effects. A conservative estimate would employ the long term creep modulus in such cases

Suggested values for modulus in MPa are given in the tables below:

Modulus (MPa)			
	Temperature °C		
	20	30	40
PVC-U & PVC-O Pressure pipes			
Short term	3000	2200	1500
Medium term	2100	1500	1100
Long term	1200	900	600
PVC-M pressure pipes			
Short term	2700	2000	1350
Medium term	1900	1350	1000
Long term	1100	800	550
PVC-U Non-pressure pipes			
Short term	3500	2500	1750
Medium term	2450	1750	1300
Long term	1400	1050	700
High Density Polyethylene			
Short term	1000	800	200
Medium term	500	400	100
Long term	200	160	80
Medium Density Polyethylene			
Short term	800	600	400
Medium term	400	300	200
Long term	160	120	80

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